

# Joyful Virtual Math Circles via Game Theory

Rodi Steinig, M.S.Ed. (she/her)

Math Renaissance

[mathrenaissance.com](http://mathrenaissance.com)



Kris, 13





Nero, 13





Zalman, 11





Obe, 11





# THE PIRATE PROBLEM

*There are 5 pirates: A, B, C, D, and E. They plunder a treasure of 100 gold coins. You are the captain, A. How do you propose splitting the treasure?*

# THE PIRATE PROBLEM: math

- Decision factors
- What is game theory?
- Backwards induction
- Dominant strategies
- Nash Equilibrium
- Definition of "rational"
- Conjectures
- Math can describe human behavior

Albert      Biraj      Callie      Daniel      Esther

A            B            C            D            E

♂            ♂            ♂            ♂            ♀

Rules

- 1) Senior - propose
- 2) All vote
- 3) Majority accepts → proposal
- 4) Majority disagree - proposal N → proposer overboard
- 5) Tie - proposer cast deciding vote

20	20	20	20	20
X	X	X	100	0 ✓
21				
60				
40	30	30	0	0
40	15	15	15	X
55	15	15	15	0

100

Decision Factors

- 1) Survival
- 2) max \$
- 3) want to see someone go
- 4) No trust

A	B	C	D	Σ
40	30	30	0	0
30	19	19	18	18
60	20	20	0	0
99				

A	B	C	D	Σ
				100
			100	0
60	40	0		
<del>100</del>	<del>0</del>	<del>0</del>		
99	12	0		
99	0	1		

Rodi Stei... →



A	B	C	D	Σ
40	30	30	0	0
30	19	19	18	18
60	20	20	0	0
99				

A	B	C	D	Σ
				100
		100	0	
60	40	0		
100	0	0		
99	12	0		
99	0	1		
95	0	5		
99	0	1		

Rodi Steir It

Rodi Stei...

Who can see what you share here?

A	B	C	D	E
—	—	—	—	—
				100 ✓
			100	0 ✓
			99	0
			99	1
			100	0
			100	0

A	B	C	D	E
—	—	—	—	—
50	25	25	0	0
99	1	1	0	0
33	33	31	1	0
95	0	25	0	0
40	15	15	15	15

•BLOG: <https://mathrenaissance.com/the-pirate-problem-applied-math-12/>

# THE PIRATE PROBLEM: reactions

OBE: “Lets  
make one of the  
decision factors  
be Who do you  
want to see  
die?”

THE PIRATE  
PROBLEM:  
reactions

NERO: “The  
money **MUST**  
be distributed  
evenly!”

THE PIRATE  
PROBLEM:  
reactions

KRIS: “Pirates  
are not  
communists!”

# THE PIRATE PROBLEM: reactions

OBE: “Is there a definite answer to this problem, or is the point of it the processing?”

# THE PRISONER'S DILEMMA

*You and your partner-in-crime get caught. The police separate you and offer you each the same deal:*

- If you both stay silent, you each get 1 year in jail.*
- If one of you confesses but the other stays silent, the one who confesses goes free and the other (silent) one gets 4 years.*
- If you both confess, you both serve 2 years.*

*What should you do?*

# THE PRISONERS DILEMMA: math

- Decision Factors
- Decision matrix
- Rational actors
- Extension questions – what if the prisoners knew the payoffs in advance – would it change things?
- Social Value Orientation – SVO – social psychology
- SPOILER: the outcome that's best for the individuals (dominant strategy) is not the outcome that's best for the group ... if people behave “rationally”
- Math problems don't have to be consistent with the real world; they only need to be consistent with themselves

Handwritten payoff matrix for the Prisoners Dilemma:

		Robert	
		silent	confess
Silent	silent	-1, -1	-4, 0
	confess	0, -4	-2, -2

The matrix is drawn with pink lines. The top row is labeled "Silent" and the bottom row "confess" on the left side. The top column is labeled "silent" and the bottom column "confess" on the right side. The name "Robert" is written in red above the columns. A red circle is drawn around the payoff (0, -4) in the bottom-right cell. A red arrow points from the text "Rodi (she/h...)" to the bottom-right cell.



# THE PRISONER'S DILEMMA: reaction

NERO: "Let's  
call the  
prisoners  
Gallavan and  
Rubbert"

# THE PRISONER'S DILEMMA: reaction

NERO: “Every  
triangle is a love  
triangle when  
you love  
triangles.”  
(Pythagorus?)

## THE PRISONER'S DILEMMA: reaction

KRIS - "These rules make absolutely no sense in the real world!"

# THE PRISONER'S DILEMMA: reaction

ZALMAN: “It’s  
cool that you  
can apply math  
to real-world  
problems.”

# CLIMATE CHANGE MODELLING

*There are only two regions in the world: North and South. The world needs to prevent the global temperature from increasing by more than 2 degrees Celsius or life as we know it will be drastically harmed. Is climate mitigation good for everyone?*

*...Mitigation will cost 6 in every region.*

*If only one region does it, it will generate a profit of 4 in both regions.*

*If both regions do it, there's a profit of 8 in both regions.*

*Should your region do it?*

<https://www.youtube.com/watch?v=iYZ5xxqY6c8>

# CLIMATE CHANGE: the math

+2°C



Decision factor  
PROFIT

profits ← NORTH

- Mitigation (prevent, reduce, make it smaller)
  - Pollute
- public good  
→ cost 6

## Rules

- If 1 country mitigates, generates profit of 4 in both  
→ spend 6, get 4 → -2
- If both mitigate, profit of 8 for both → spend 6  
→ +2

	pollute	mitigate
pollute	0	4
mitigate	-2	2

Diagonal labels: pollute, pollute

Annotations: Rodi (she/h...) SOUTH, Rodi Steinig

→ Nash equilibrium: both pollute

# CLIMATE CHANGE: reaction

Climate change  
prisoner's dilemma

good model

hopeless?

unsure

WAR?

punish each other?

sanctions?

binding agreements?

look at long term

~~bad model~~

- Not all countries equal
- ethical leaders
- more complex, other decision factor  
Rodi Stei... →
- public good -  
govts + companies  
can collaborate
- mitigate lower cost
- public pressure
- decision makers are  
not robots



- KRIS, ZALMAN, NERO: The Prisoner's Dilemma is a bad model for Climate Talks.
- OBE: It's hopeless.

EMOTIONAL  
RELIEF:  
Would You  
Rather?

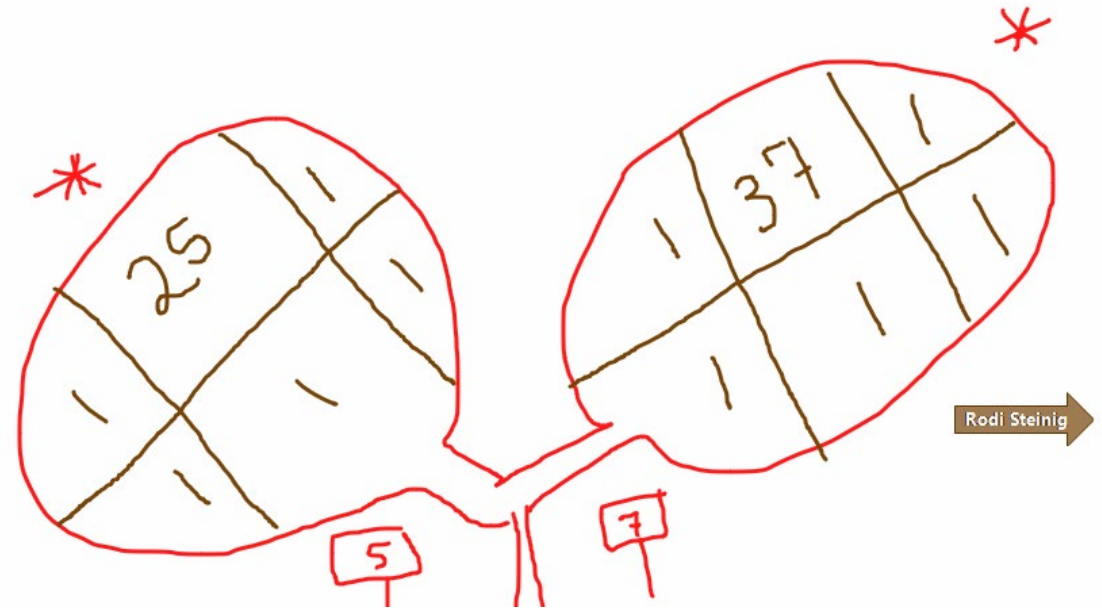
- *“You are exploring a land populated by hydrophobic vicious animals. You are safely wading in a one-foot deep stream when you come to a fork in it. Each branch leads to a different pond. Each pond has a helicopter on the other side of it that can transport you to safety. A sign at the fork tells you that the pond on one side has an average depth of 5 feet, and the other is 7. Oh, and did I mention that you can’t swim?”*

# WOULD YOU RATHER? math

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...central tendency version dispersion

- You have to make assumptions
- You have to make some guesses
- Uncertainty!



# Function Machines: OTHER EMOTIONAL RELIEF

Definition of a function

Domain & range

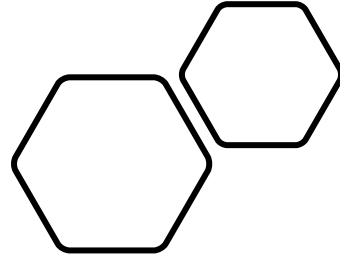
Conditional functions

Equivalent expressions

Non-numeric/algebraic functions

Truth values

TRAGEDY  
(or non-  
tragedy?)  
OF THE  
COMMONS



Are shared (“common  
pool”) resources  
inevitably destroyed?

# TRAGEDY OF THE COMMONS: math

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- traffic congestion  
(roads)

- health (antibiotics)

- irrigation

- Earth ("Don't  
Look Up")

- climate

- international relations

- peace

- fisheries\*

- ocean

- oil

- gaming (characters)

- grazing lands

- prisoners dilemma

- groundwaters

- land

# TRAGEDY OF THE NON-COMMONS: math



Ostrim  
Maine  
Indonesia  
Nepal  
Kenya  
Decision  
makers  
physically  
close to  
the  
resource



Maps of Africa  
Africa Map

# TRAGEDY OF THE COMMONS: reaction

Elinor Ostrom



Ostrom in 2009

**Born** Elinor Claire Awan  
August 7, 1933  
[Los Angeles, California,  
United States](#)

**Died** June 12, 2012 (aged 78)  
[Bloomington, Indiana,](#)

Nobel Prize



**Awarded for** Contributions that have  
conferred the greatest benefit  
to Mankind in the areas of

OUR HERO!



# Math Circle Game

Form description



"NAME" (can be fake as long as not triggering)



Short answer



Short answer text



Required



Which point distribution do you prefer for the game?

A: You get 550, other person gets 300

B: You get 500, other person gets 100

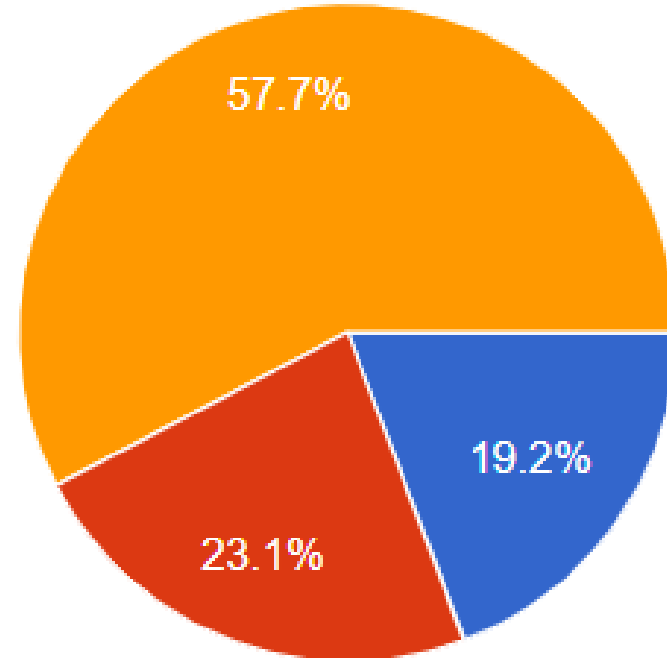
C: You get 500, other person gets 500

# TRIPLE DOMINANCE GAME: reaction

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Which point distribution do you prefer for the game?

26 responses



- A: You get 550, other person gets 300
- B: You get 500, other person gets 100
- C: You get 500, other person gets 500

TRIPLE  
DOMINANCE  
GAME: math

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Decision theory versus game theory

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Social psychology

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Data collection

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Graphing

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Positing conjectures

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Communication

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# BUTTERED TOAST PROBLEM

Costs: 1 tickets

Win: 3 tickets

Lose: nothing

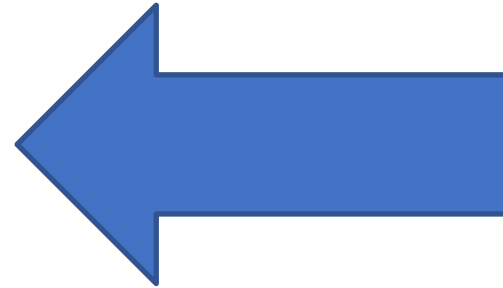
Chance: 50% win

Q: should you play?

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Wondering?

- How many tickets have?
- What are tickets worth?
- What is the opportunity cost? Other games



# The Buttered Toast Game/Problem

## Given Info

Game

Cost: 1 ticket

Win: get 3 tix

Chances: 50%

Lose: nothing

## Questions

Should you play?

How many tix do

you have? → 5

what is the game? → buttered toast

How much are tix worth? To you? → \$1 per ticket

"Opportunity Cost"

- other games? → No, locked up

- better games? → in basement

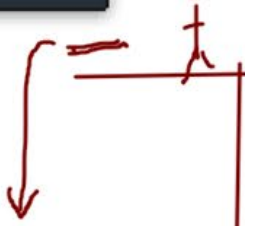
- better payoffs? → of ~~someone's~~ (LESHY'S)

- \$, tix

- ("utility")

of someone's cabin w/ high ceiling

## Assumption



win

lose



LESHY

# BUTTERED TOAST PROBLEM: math

<u>play</u>	<u>total invest</u>	<u>outcome</u>	<u>profit</u>
1 time	1	L	1: (-1)
2 times	2	L <sub>0</sub> W <sub>3</sub>	2-3 = 1
3 times	3	L <sub>0</sub> W <sub>3</sub> L <sub>0</sub>	3-3 = 0
4 times	4	L <sub>0</sub> W <sub>3</sub> L <sub>0</sub> W <sub>3</sub>	6-4 = 2
5	5	L <sub>0</sub> L <sub>0</sub> L <sub>0</sub> W <sub>3</sub> W <sub>3</sub>	6-5 = 1
6			
7			
8			

$y = -x^2 + 4x$   
 $z = -x^2 + 4x$   
 $z = -x^2 + 4x$   
 Rodi Steinig → ?  
 → could run out

- Probability
- Opportunity cost
- Utility
- Looking for patterns
- (Expected value)

RATIONAL  
ACTORS:  
COIN  
TOSSING

Toss coin

Tails: 50% ← if you toss a certain way,  
open space, randomly,  
not trying to influence

1 2 3 4 5 6 7 8 9 10  
T T T T T T T T T

Probability of getting T → 11<sup>th</sup> throw? 50%

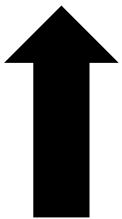


# COIN TOSSING: math

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- Behavioral economics versus neoclassical economics (what we actually do versus what we should do)
- Law of Large Numbers
- Gambler's Fallacy

	A	B	C	D	E
1		<b>Person</b>	<b>Number of tosses</b>	<b>Number of tails</b>	<b>Percent tails</b>
2		J	1	1	100%
3		O	1	1	100%
4		Z	1	0	0%
5		N	1	1	100%
6		J	2	2	100%
7		O	2	0	0%
8		Z	2	1	50%
9		N	2	2	100%
10		J	3	2	67%
11		O	3	2	67%
12		Z	3	1	33%
13		N	3	0	0%
14		J	4	1	25%
15		O	4	1	25%
16		Z	4	4	100%

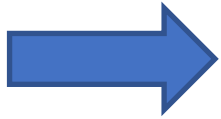


17		N	4	2	50%
18		J	5	2	40%
19		O	5	2	40%
20		Z	5	2	40%
21		N	5	0	0%
22		J	6	5	83%
23		O	6	2	33%
24		Z	6	3	50%
25		N	6	2	33%
26		J	7	3	43%
27		O	7	3	43%
28		Z	7	5	71%
29		N	7	2	29%
30		J	8	2	25%
31		O	8	3	38%
32		Z	8	6	75%
33		N	8	4	50%
34		J	9	5	56%
35		O	9	4	44%
36		Z	9	3	33%
37		N	9	2	22%
38		J	10	6	60%
39		O	10	5	50%
40		Z	10	7	70%
41		N	10	5	50%
42		J	11	6	55%
43		O	11	7	64%
44		Z	11	8	73%
45		N	11	3	27%
46		J	12	7	58%
47		O	12	3	25%
48		Z	12	6	50%
49		N	12	6	50%



1		<b>Person</b>	<b>Number of tosses</b>	<b>Number of tails</b>	<b>Percent tails</b>
44		Z	11	8	73%
45		N	11	3	27%
46		J	12	7	58%
47		O	12	3	25%
48		Z	12	6	50%
49		N	12	6	50%
50		<b>TOTAL</b>	<b>312</b>	<b>150</b>	<b>48%</b>

	A	B	C	D	E
1		<b>Person</b>	<b>Number of tosses</b>	<b>Number of tails</b>	<b>Percent tails</b>
44		Z	11	8	73%
45		N	11	3	27%
46		J	12	7	58%
47		O	12	3	25%
48		Z	12	6	50%
49		N	12	6	50%
50		TOTAL	312	150	48%
51					
52		guess	1,000	500	50%



	A	B	C	D	E
1		<b>Person</b>	<b>Number of tosses</b>	<b>Number of tails</b>	<b>Percent tails</b>
44		Z	11	8	73%
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49		N	12	6	50%
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52		guess	1,000	500	50%
53		guess	50,000,000	25000000	50%



	A	B	C	D	E
1		<b>Person</b>	<b>Number of tosses</b>	<b>Number of tails</b>	<b>Percent tails</b>
44		Z	11	8	73%
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46		J	12	7	58%
47		O	12	3	25%
48		Z	12	6	50%
49		N	12	6	50%
50		TOTAL	312	150	48%
51					
52		guess	1,000	500	50%
53		guess	50,000,000	25000000	50%
54		guess	2,020	1010	50%



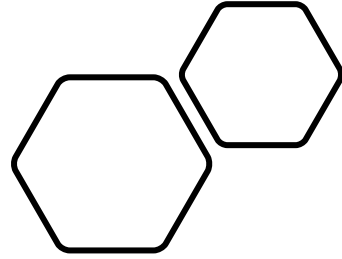
## Conjectures

Larger margin of error for small number of tosses

There are ways to make the flips not influenced by skill etc

The more shots you take, the more shots you take. If you keep doing it, it will be more even.

# DIMINISHING MARGINAL UTILITY OF MONEY



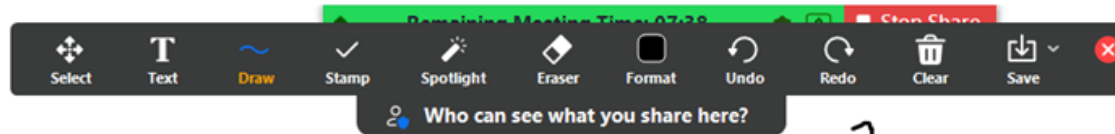
Would you eat  
mud for a million  
dollars?



# WOULD YOU EAT MUD: reaction



- does it kill you?
- how much?



<u>offer</u>	<u>eat?</u>
\$1,000,000	Y, Y, Y, N
\$1,000,000,000	Y, Y, Y, N
\$1,000,000,000,000	Y, Y, Y, <del>N</del> N

MIN threshold?

\$10,000  
\$75,000  
\$1,000  
\$300,000  
\$

# WOULD YOU EAT MUD?

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- CONTEXT MATTERS!



Rodi Steinig

have: \$1,000,000,000

eat?: \$1,000,000    N, N, N, N

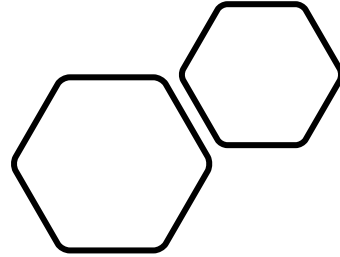
source  
of  
income?

YES

Salary: 1,000,000 / year

"Diminishing  
marginal  
utility of  
money"

# “NEWCOMB’S” PROBLEM (v0.0)



You enter a circus tent and are shown two boxes. One is clear and contains \$1,000. The other is opaque and contains either nothing or \$1,000,000. You can choose one or the other. Which do you choose?

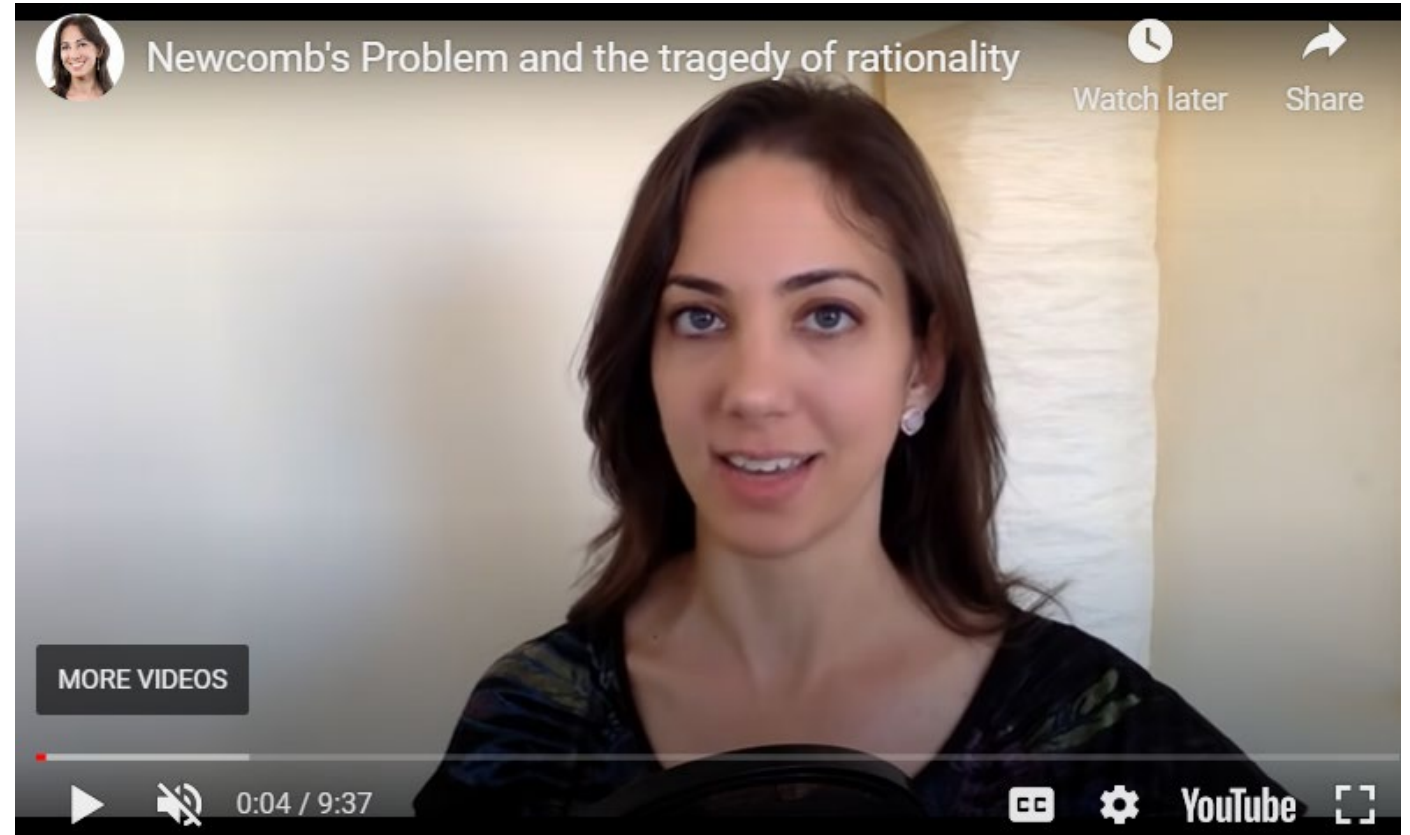
# “NEWCOMB’S PROBLEM” v0.0, reaction

ZALMAN: “This question seems really simple. What else is going on?”

## NEWCOMB'S PROBLEM – the real thing!

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You enter a circus tent and are shown two boxes. One is clear and contains \$1,000. The other is opaque and contains either nothing or \$1,000,000. You can choose one or you can choose both. And...



Julia Galef, philosopher and skeptic

- ... as you walked into the tent, there was a perfect predictor who could tell in advance with 100% certainty which box you would pick. If the predictor predicts that you would take just the opaque box, the circus tent woman put \$1,000,000 in that box, but if the predictor predicts that you would take both, the circus tent woman puts nothing in the opaque box.

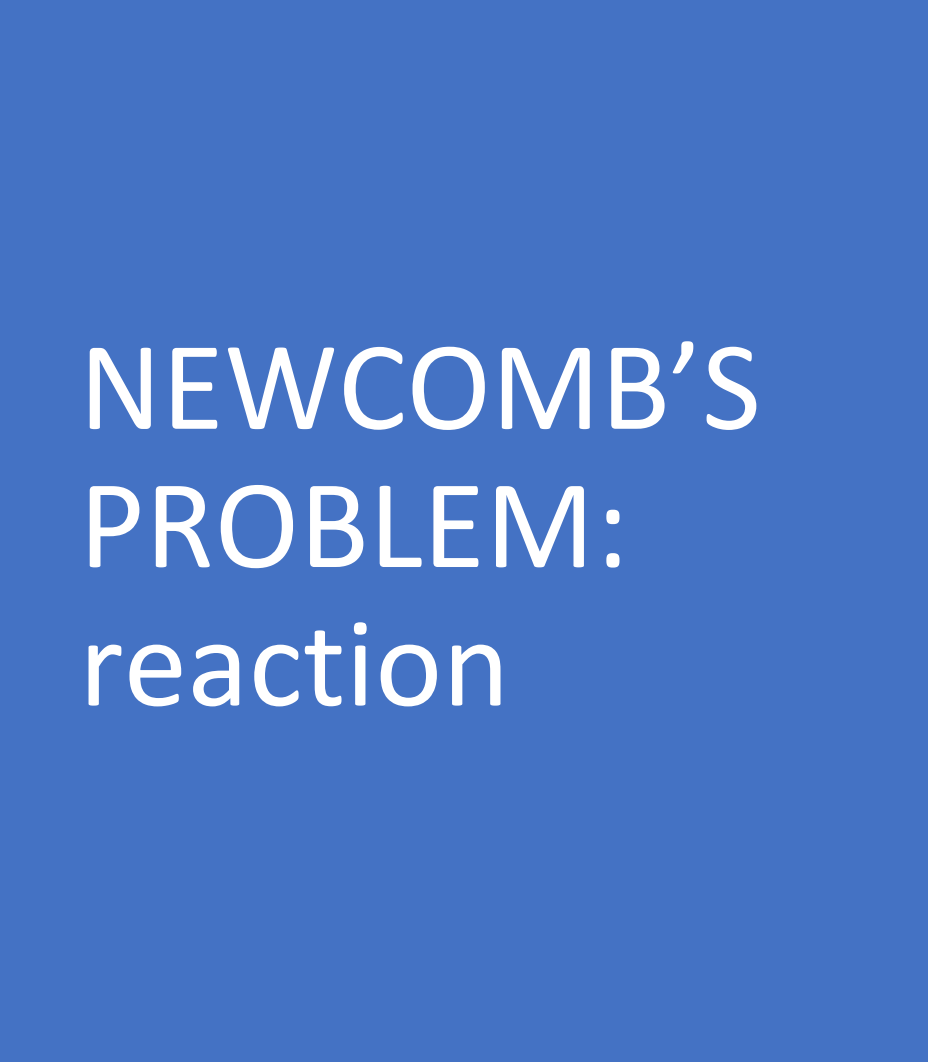
Newcomb's Problem and the tragedy of rationality

NEWCOMB'S PROBLEM: Do you take just the opaque box, or both?

The diagram illustrates the Newcomb's Problem. On the left, a cartoon character with gears in his head represents the predictor. Two dashed arrows labeled "Prediction" point from the predictor to two thought bubbles: "Will one-box" and "Will two-box". From the "Will one-box" bubble, a dashed arrow labeled "\$1 mil" points to an opaque box with a question mark on its side. From the "Will two-box" bubble, a dashed arrow labeled "Nothing" points to the same opaque box. To the right of the opaque box is a transparent box labeled "\$1000". Above the boxes, two hands are shown dropping coins into the opaque box. At the bottom left, there is a "MORE VIDEOS" button, and at the bottom right, there is a "Full screen (f)" button.


NEWCOMB'S  
PROBLEM:  
math

- What is a skeptic?
- Causal versus experiential decision theory
- Choice
- Free will

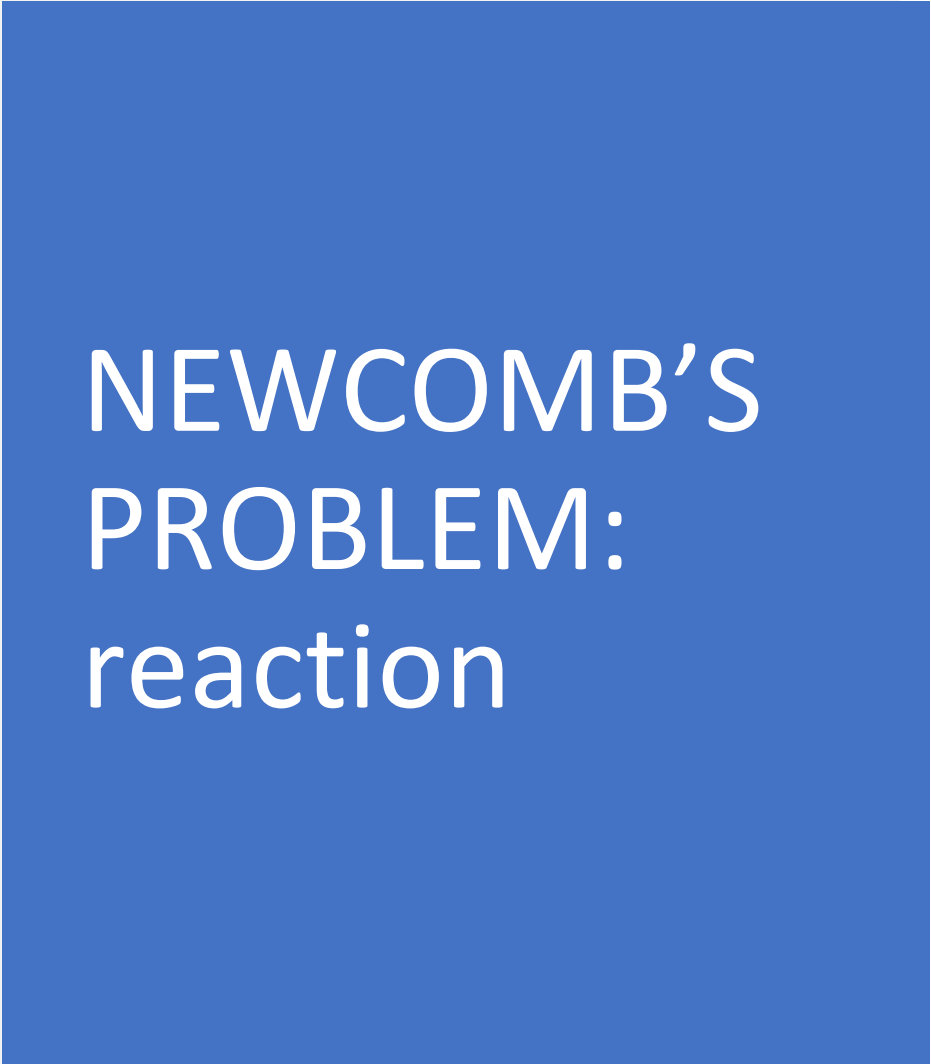


NEWCOMB'S  
PROBLEM:  
reaction

OBE: "I don't know  
why you would pick  
both boxes!"

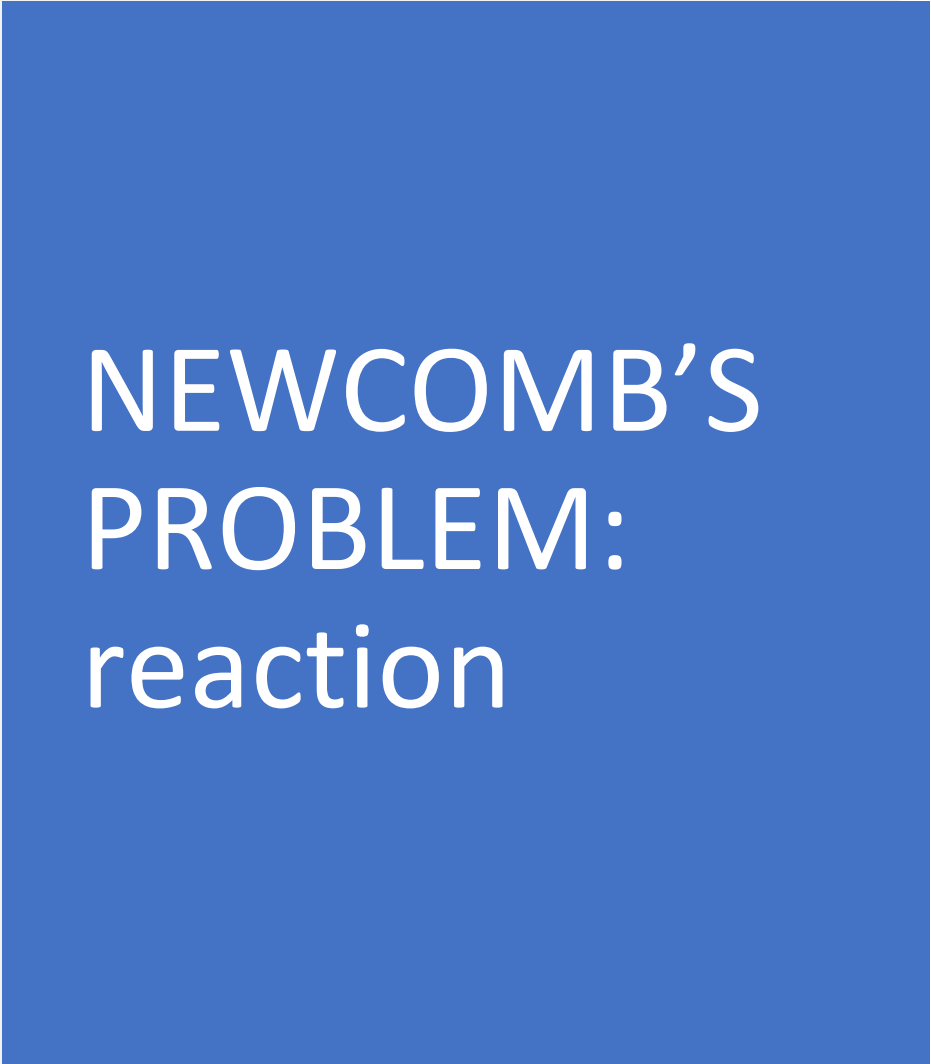






NEWCOMB'S  
PROBLEM:  
reaction

KRIS: "This is frying  
my brain!"




NEWCOMB'S  
PROBLEM:  
reaction

ZALMAN – “It seems  
really simple until...”





NEWCOMB'S  
PROBLEM:  
reaction



NERO: “Is this  
something a  
mathematician  
would wear?”



STABLE  
MATCHING  
v1.0 (KIDNEY  
EXCHANGE)

If you needed a kidney  
and were offered one  
from a 90-year-old man,  
would you take it?

# STABLE MATCHING v1.0: math

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“KIDNEY EXCHANGE”

- Decision Factors

RECIPIENTS of Kidneys

- Age
- wait time
- Risky
- Trust in doctor
- Race

DONORS of kidneys

- HLA ~~match~~ mismatch
- KIR genotype
- Age
- Gender?
- Relation

STABLE  
MATCHING  
v2.0  
(SCHOOL  
CHOICE)

STEP 1: Create a character whose name begins with one letter that is in your name...



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STEP 2: Rank (in writing) your character's preferences for the following schools:

**Wizard School**

**Flying School**

**Diplomat School**

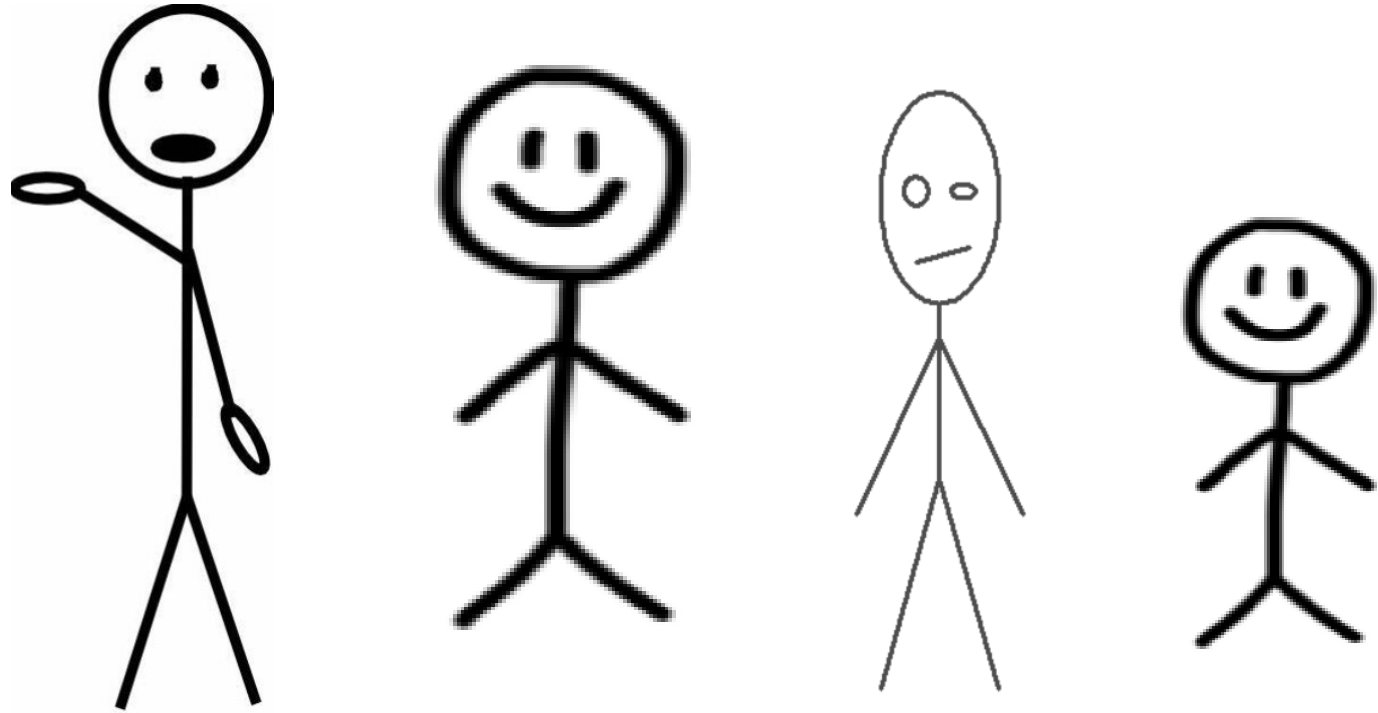
**Art School**



STEP 3: While you were ranking the schools, the schools were ranking “you.” Let’s apply the Gale-Shapley algorithm to obtain a stable match!

	<b>SCHOOLS' PREFERENCES</b>			
	<b><u>Wizard School</u></b>	<b><u>Flying School</u></b>	<b><u>Diplomat School</u></b>	<b><u>Art School</u></b>
1st choice	O	N	O	O
2nd choice	Z	Z	N	N
3rd choice	K	O	Z	Z
4th choice	N	K	K	K

# STABLE MATCHING: reaction



	<b>SCHOOLS' PREFERENCES</b>			
	<b>Wizard School</b>	<b>Flying School</b>	<b>Diplomat School</b>	<b>Art School</b>
1st choice	O	N	O	O
2nd choice	Z	Z	N	N
3rd choice	K	O	Z	Z
4th choice	N	K	K	K



THANK YOU!

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