Joyful Virtual Math Circles via Game Theory

Rodi Steinig, M.S.Ed. (she/her)

Math Renaissance

mathrenaissance.com

Kris, 13

Nero, 13

Zalman, 11

Obe, 11

THE PIRATE PROBLEM

There are 5 pirates: A,B,C,D, and E. They plunder a treasure of 100 gold coins. You are the captain, A. How do you propose splitting the treasure?

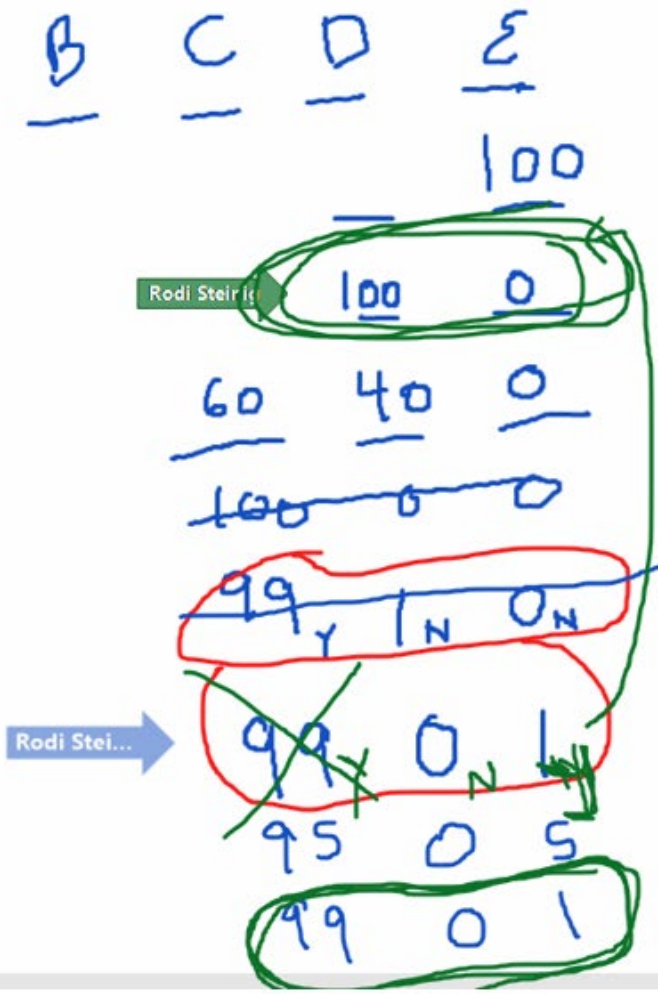
THE PIRATE PROBLEM: math

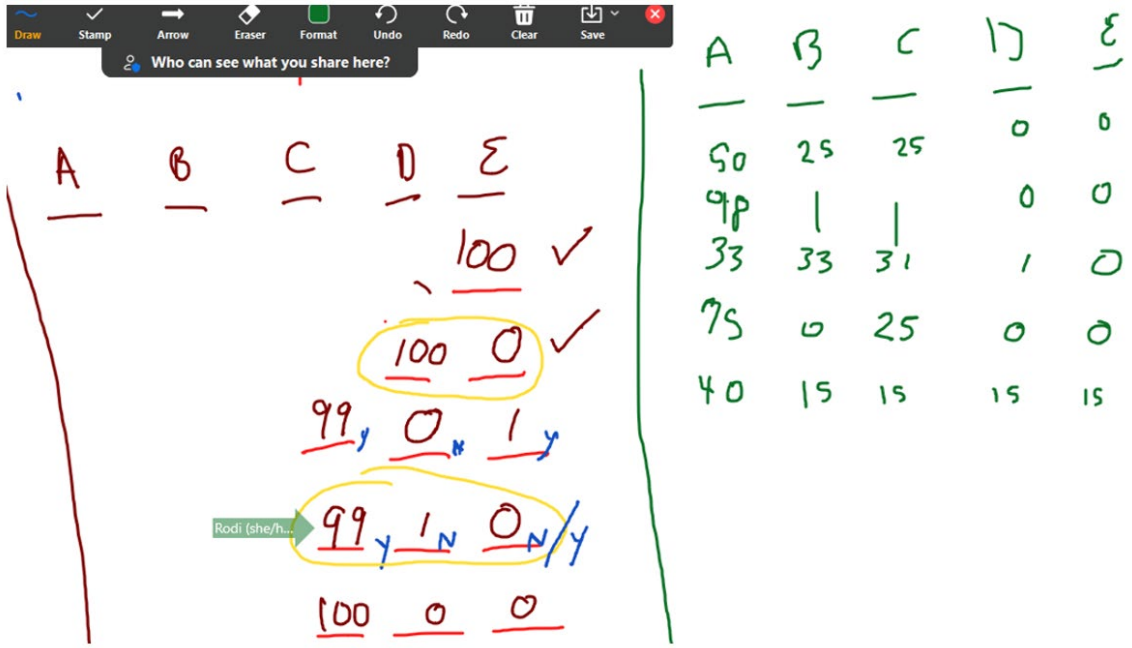
- Decision factors
- What is game theory?
- Backwards induction
- Dominant strategies
- Nash Equillibrium
- Definition of “rational”
- Conjectures
- Math can describe human behavior

A	B	C	D	E
40	30	30	0	0
30	19	19	18	18
60	20	20	2	2
99				

A	B	C	D	E
				100
			—	—
			100	0
		60	40	0
		~~100~~	0	0
		99 Y	1 N	0 N
Rodi Stei...		99	0	1

A	B	C	D	E
40	30	30	0	0
30	19	19	18	18
60	20	20	0	0
99				





•BLOG: https://mathrenaissance.com/the-pirate-problem-applied-math-12/

THE PIRATE PROBLEM: reactions

OBE: “Lets make one of the decision factors be Who do you want to see die?”

THE PIRATE PROBLEM: reactions

NERO: “The money MUST be distributed evenly!”

THE PIRATE PROBLEM: reactions

KRIS: “Pirates are not communists!”

THE PIRATE PROBLEM: reactions

OBE: “Is there a definite answer to this problem, or is the point of it the processing?”

THE PRISONER'S DILEMNA

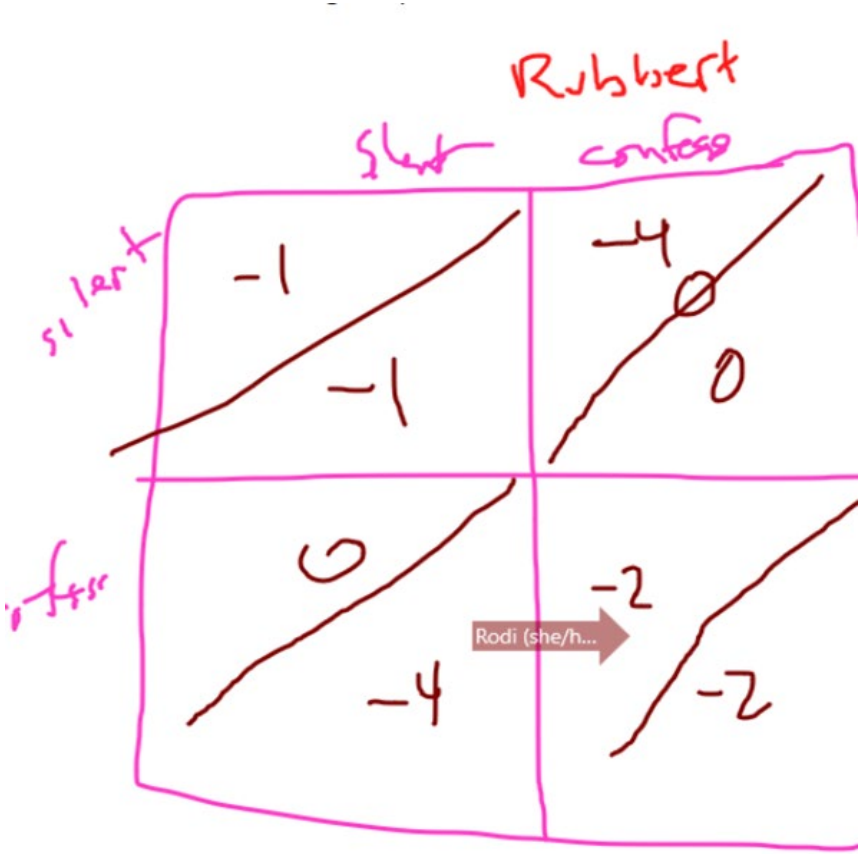
You and your partner-in-crime get caught. The police separate you and offer you each the same deal:

- *If you both stay silent, you each get 1 year in jail.*
- *If one of you confesses but the other stays silent, the one who confesses goes free and the other (silent) one gets 4 years.*
- *If you both confess, you both serve 2 years.*

What should you do?

THE PRISONERS DILEMNA: math

- Decision Factors
- Decision matrix
- Rational actors
- Extension questions – what if the prisoners knew the payoffs in advance – would it change things?
- Social Value Orientation – SVO – social psychology
- SPOILER: the outcome that's best for the individuals (dominant strategy) is not the outcome that's best for the group … if people behave "rationally"
- Math problems don't have to be consistent with the real world; they only need to be consistent with themselves



THE PRISONER'S DILEMNA: reaction

NERO: "Let's call the prisoners Gallavan and Rubbert"

THE PRISONER'S DILEMNA: reaction

NERO: "Every triangle is a love triangle when you love triangles." (Pythagorus?)

THE PRISONER'S DILEMNA: reaction

KRIS - "These rules make absolutely no sense in the real world!"

THE PRISONER'S DILEMNA: reaction

ZALMAN: "It's cool that you can apply math to real-world problems."

CLIMATE CHANGE MODELLING

There are only two regions in the world: North and South. The world needs to prevent the global temperature from increasing by more than 2 degrees Celsius or life as we know it will be drastically harmed. Is climate mitigation good for everyone?

…Mitigation will cost 6 in every region.

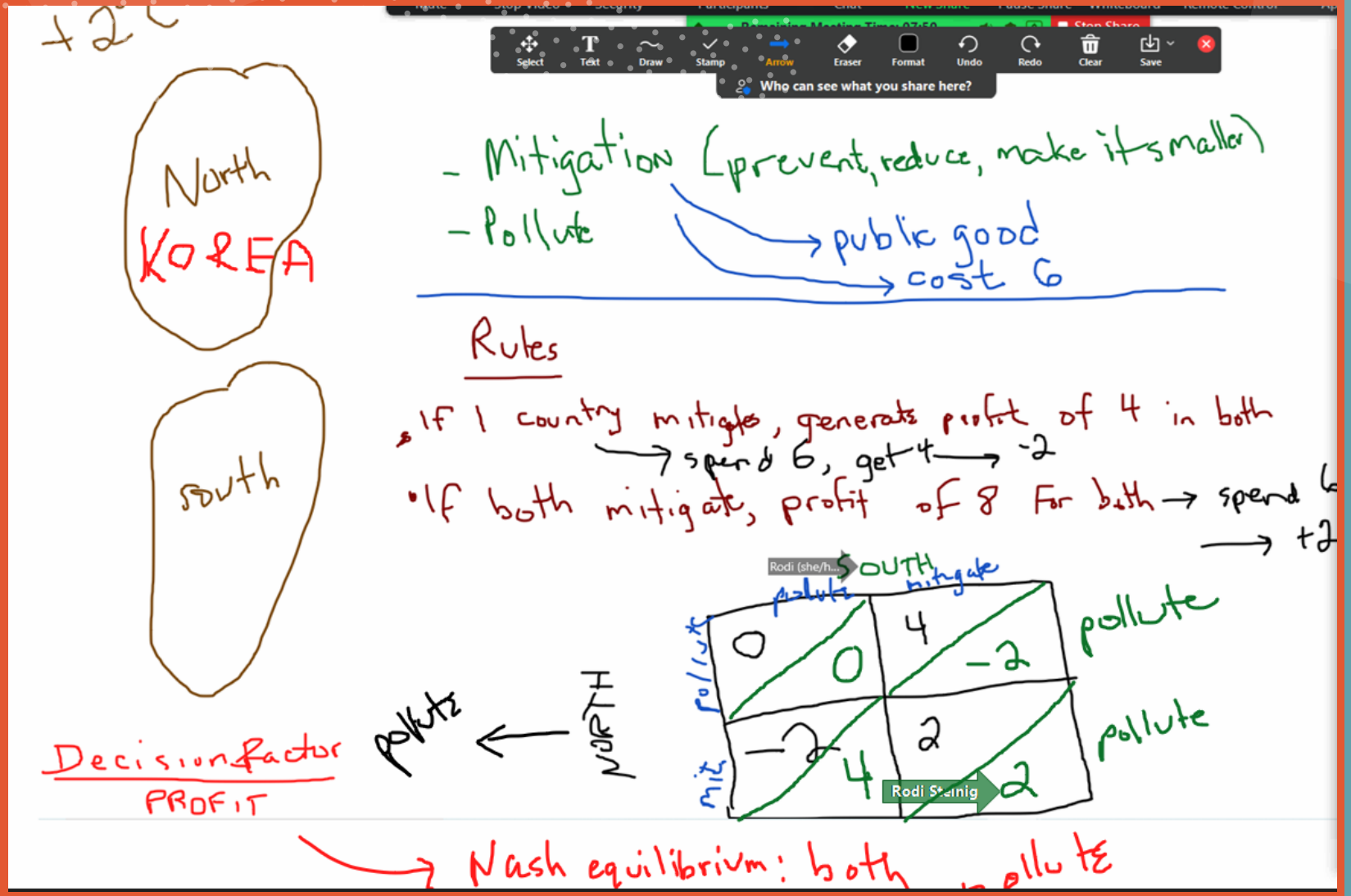
If only one region does it, it will generate a profit of 4 in both regions.

If both regions do it, there's a profit of 8 in both regions.

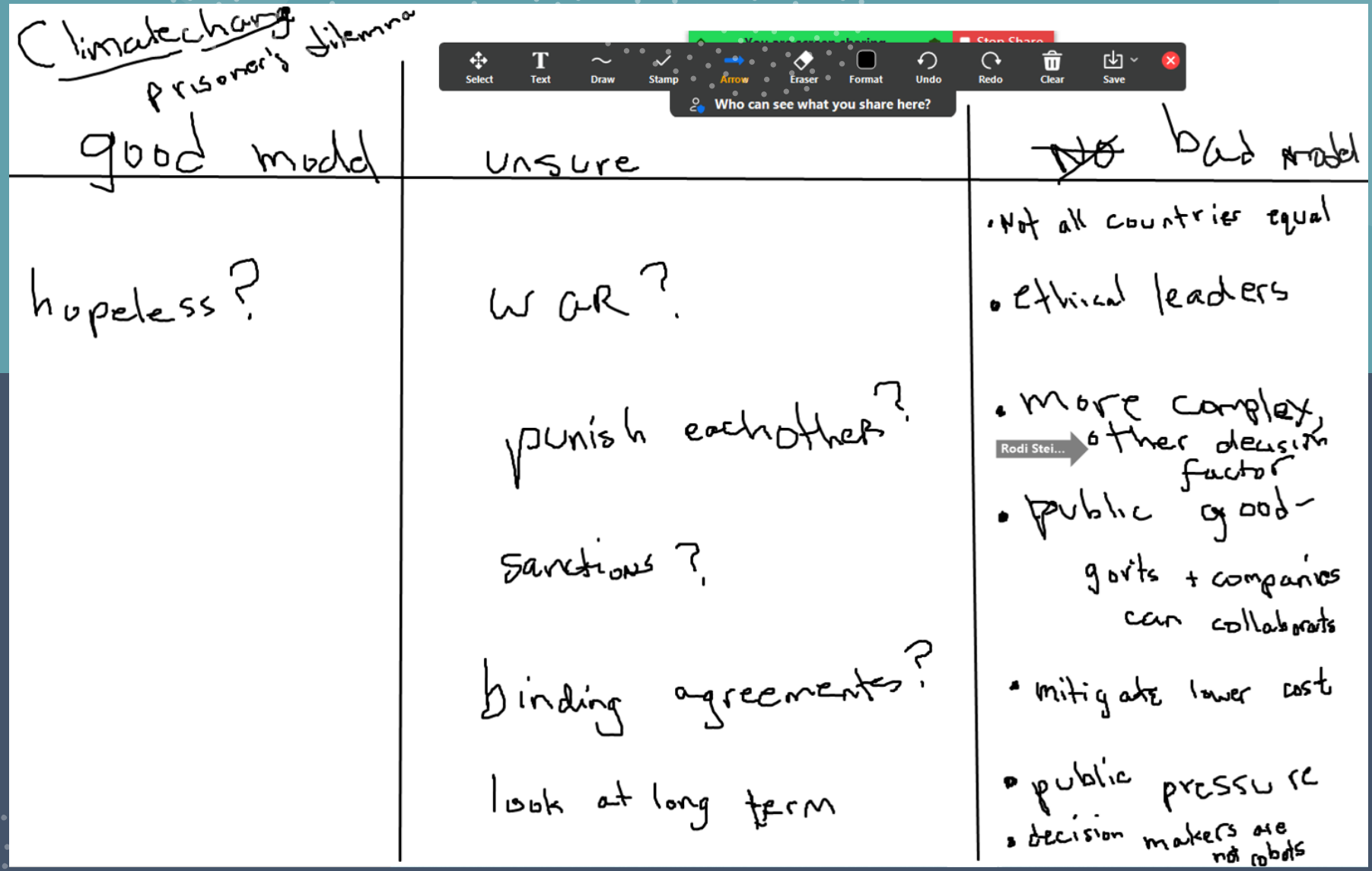
Should your region do it?

https://www.youtube.com/watch?v=iYZ5xxqY6c8

CLIMATE CHANGE: the math



CLIMATE CHANGE: reaction



- KRIS, ZALMAN, NERO: The Prisoner’s Dilemma is a bad model for Climate Talks.

- OBE: It’s hopeless.

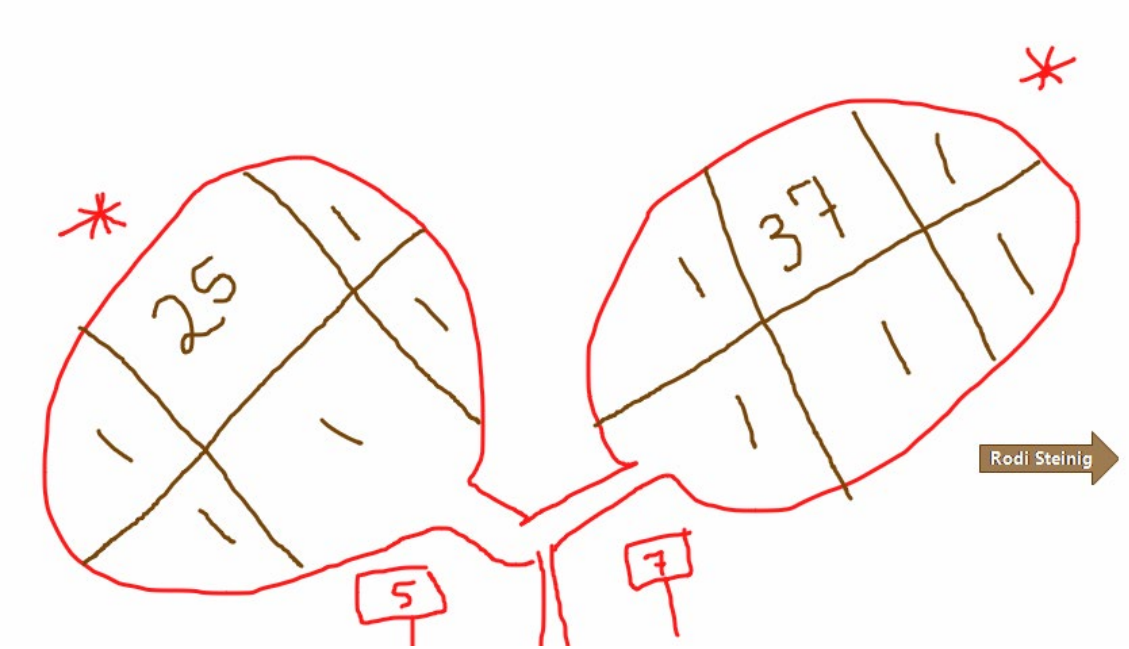
EMOTIONAL RELIEF: Would You Rather?

- *“You are exploring a land populated by hydrophobic vicious animals. You are safely wading in a one-foot deep stream when you come to a fork in it. Each branch leads to a different pond. Each pond has a helicopter on the other side of it that can transport you to safety. A sign at the fork tells you that the pond on one side has an average depth of 5 feet, and the other is 7. Oh, and did I mention that you can’t swim?”*

WOULD YOU RATHER? math

…central tendency version dispersion

- You have to make assumptions
- You have to make some guesses
- Uncertainty!



Function Machines: OTHER EMOTIONAL RELIEF

Definition of a function

Domain & range

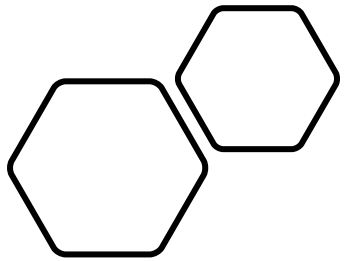
Conditional functions

Equivalent expressions

Non-numeric/algebraic functions

Truth values

TRAGEDY (or non-tragedy?) OF THE COMMONS



Are shared (“common pool”) resources inevitably destroyed?

TRAGEDY OF THE COMMONS: math

- traffic congestion (roads)
- health (antibiotics)
- irrigation
- Earth ("Don't Look Up")
- climate
- international relations
- peace
- fisheries ★
- ocean
- oil
- gaming (characters)
- grazing lands
- prisoners dilemma
- groundwaters
- land

TRAGEDY OF THE NON-COMMONS: math

TRAGEDY OF THE COMMONS: reaction

Elinor Ostrom

Ostrom in 2009

Born	Elinor Claire Awan August 7, 1933 Los Angeles, California, United States
Died	June 12, 2012 (aged 78) Bloomington, Indiana,

Nobel Prize

Awarded for	Contributions that have conferred the greatest benefit to Mankind in the areas of

OUR HERO!

TRIPLE DOMINANCE GAME

Math Circle Game

Form description

"NAME" (can be fake as long as not triggering)

Short answer

Short answer text

Required

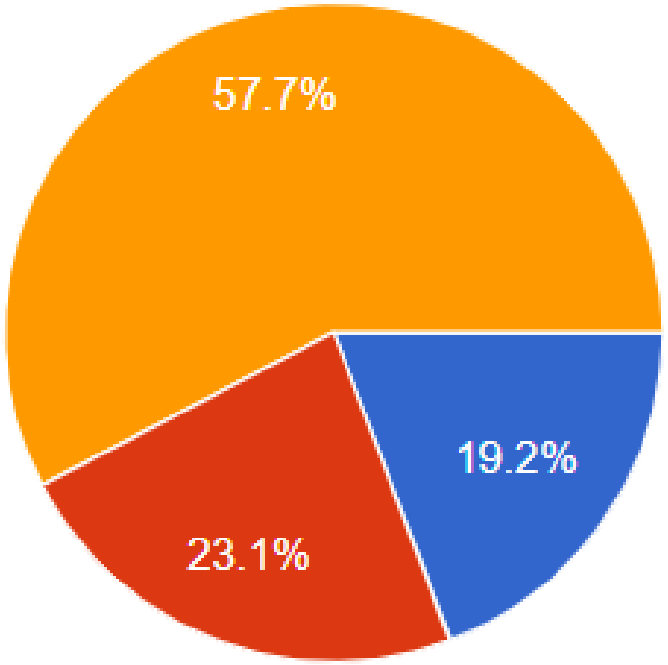
Which point distribution do you prefer for the game?

- A: You get 550, other person gets 300
- B: You get 500, other person gets 100
- C: You get 500, other person gets 500

TRIPLE DOMINANCE GAME: reaction

Which point distribution do you prefer for the game?

26 responses



A: You get 550, other person gets 300
B: You get 500, other person gets 100
C: You get 500, other person gets 500

TRIPLE DOMINANCE GAME: math

- Decision theory versus game theory
- Social psychology
- Data collection
- Graphing
- Positing conjectures
- Communication

BUTTERED TOAST PROBLEM

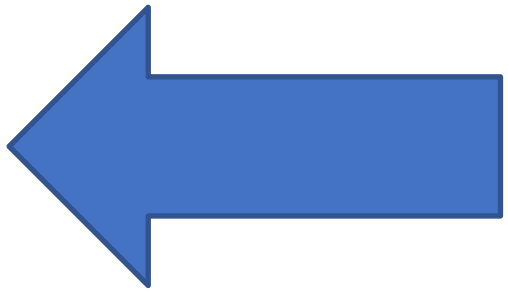
Costs: 1 tickets

Win: 3 tickets

Lose: nothing

chance: 50% win

Q: should you play?



Wondering?

- How many tix have?
- what are tickets worth?
- What is the opportunity cost? other games

The Buttered Toast Game/Problem

Select | Text | Draw | Stamp | Arrow | Eraser | Format | Undo | Redo | Clear | Save

Who can see what you share here?

Given Info

Game
Cost : 1 ticket
Win: get 3 tix
Chances: 50%
Lose: nothing

Questions

Should you play?

How many tix do you have? → 5

What is the game? → buttered toast

How much are tix worth? To you? → $1 per ticket

"Opportunity Cost"
- other games? → No, locked up in basement of ~~someone's~~ (LESHY'S) cabin w/ high ceiling
- better games?
- better payoffs?
  - $/tix
  - ("utility")

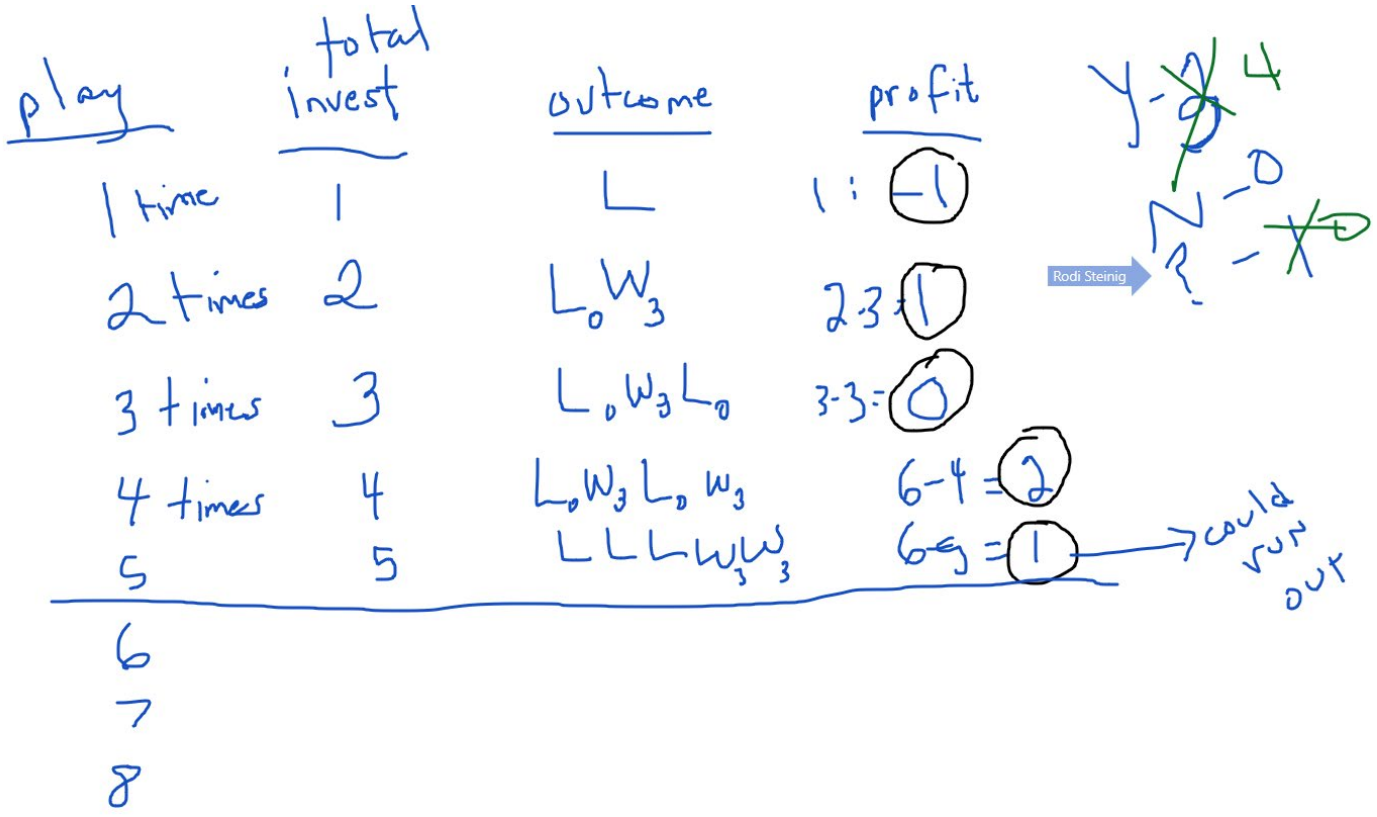
Assumption

win

lose

LESHY

BUTTERED TOAST PROBLEM: math



- Probability
- Opportunity cost
- Utility
- Looking for patterns
- (Expected value)

RATIONAL ACTORS: COIN TOSSING

Toss coin

Tails: 50%. ← if you toss a certain way, open space, randomly, not trying to influence

1 2 3 4 5 6 7 8 9 10

T T T T T T T T T T

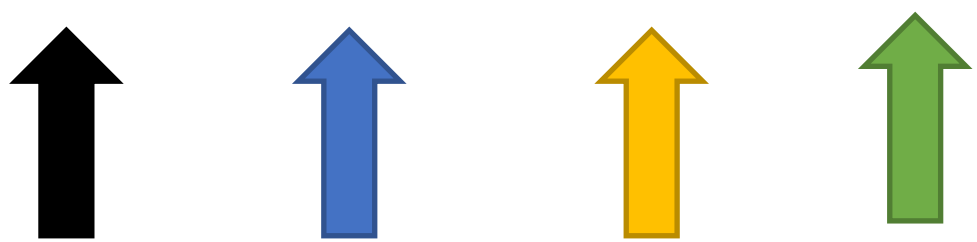
Probability of getting T on 11th throw? 50%

Rodi Steinig

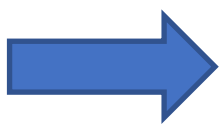
COIN TOSSING: math

- Behavioral economics versus neoclassical economics (what we actually do versus what we should do)
- Law of Large Numbers
- Gambler's Fallacy

	A	B	C	D	E
1		**Person**	**Number of tosses**	**Number of tails**	**Percent tails**
2		J	1	1	100%
3		O	1	1	100%
4		Z	1	0	0%
5		N	1	1	100%
6		J	2	2	100%
7		O	2	0	0%
8		Z	2	1	50%
9		N	2	2	100%
10		J	3	2	67%
11		O	3	2	67%
12		Z	3	1	33%
13		N	3	0	0%
14		J	4	1	25%
15		O	4	1	25%
16		Z	4	4	100%



17	N	4	2	50%
18	J	5	2	40%
19	O	5	2	40%
20	Z	5	2	40%
21	N	5	0	0%
22	J	6	5	83%
23	O	6	2	33%
24	Z	6	3	50%
25	N	6	2	33%
26	J	7	3	43%
27	O	7	3	43%
28	Z	7	5	71%
29	N	7	2	29%
30	J	8	2	25%
31	O	8	3	38%
32	Z	8	6	75%
33	N	8	4	50%
34	J	9	5	56%
35	O	9	4	44%
36	Z	9	3	33%
37	N	9	2	22%
38	J	10	6	60%
39	O	10	5	50%
40	Z	10	7	70%
41	N	10	5	50%
42	J	11	6	55%
43	O	11	7	64%
44	Z	11	8	73%
45	N	11	3	27%
46	J	12	7	58%
47	O	12	3	25%
48	Z	12	6	50%
49	N	12	6	50%



1	Person	Number of tosses	Number of tails	Percent tails
44	Z	11	8	73%
45	N	11	3	27%
46	J	12	7	58%
47	O	12	3	25%
48	Z	12	6	50%
49	N	12	6	50%
50	TOTAL	312	150	48%

	A	B	C	D	E
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44		Z	11	8	73%
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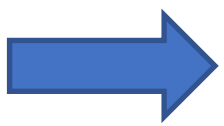
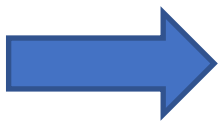
47		O	12	3	25%
48		Z	12	6	50%
49		N	12	6	50%
50		TOTAL	312	150	48%
51					
52		guess	1,000	500	50%





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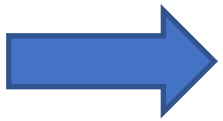
52		guess	1,000	500	50%
53		guess	50,000,000	25000000	50%



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47		O	12	3	25%
48		Z	12	6	50%
49		N	12	6	50%
50		TOTAL	312	150	48%
51					
52		guess	1,000	500	50%
53		guess	50,000,000	25000000	50%
54		guess	2,020	1010	50%

Conjectures
Larger margin of error for small number of tosses
There are ways to make the flips not influenced by skill etc
The more shots you take, the more shots you take. If you keep doing it, it will be more even.

DIMINISHING MARGINAL UTILITY OF MONEY

Would you eat mud for a million dollars?

WOULD YOU EAT MUD: reaction

- does it kill you?
- how much?

Select Text Draw Stamp Spotlight Eraser Format Undo Redo Clear Save

Who can see what you share here?

offer	eat?
$1,000,000	Y, Y, Y, N
$1,000,000,000	Y, Y, Y, N
$1,000,000,000	Y, Y, Y, N

min threshold?

$10,000

$75,000

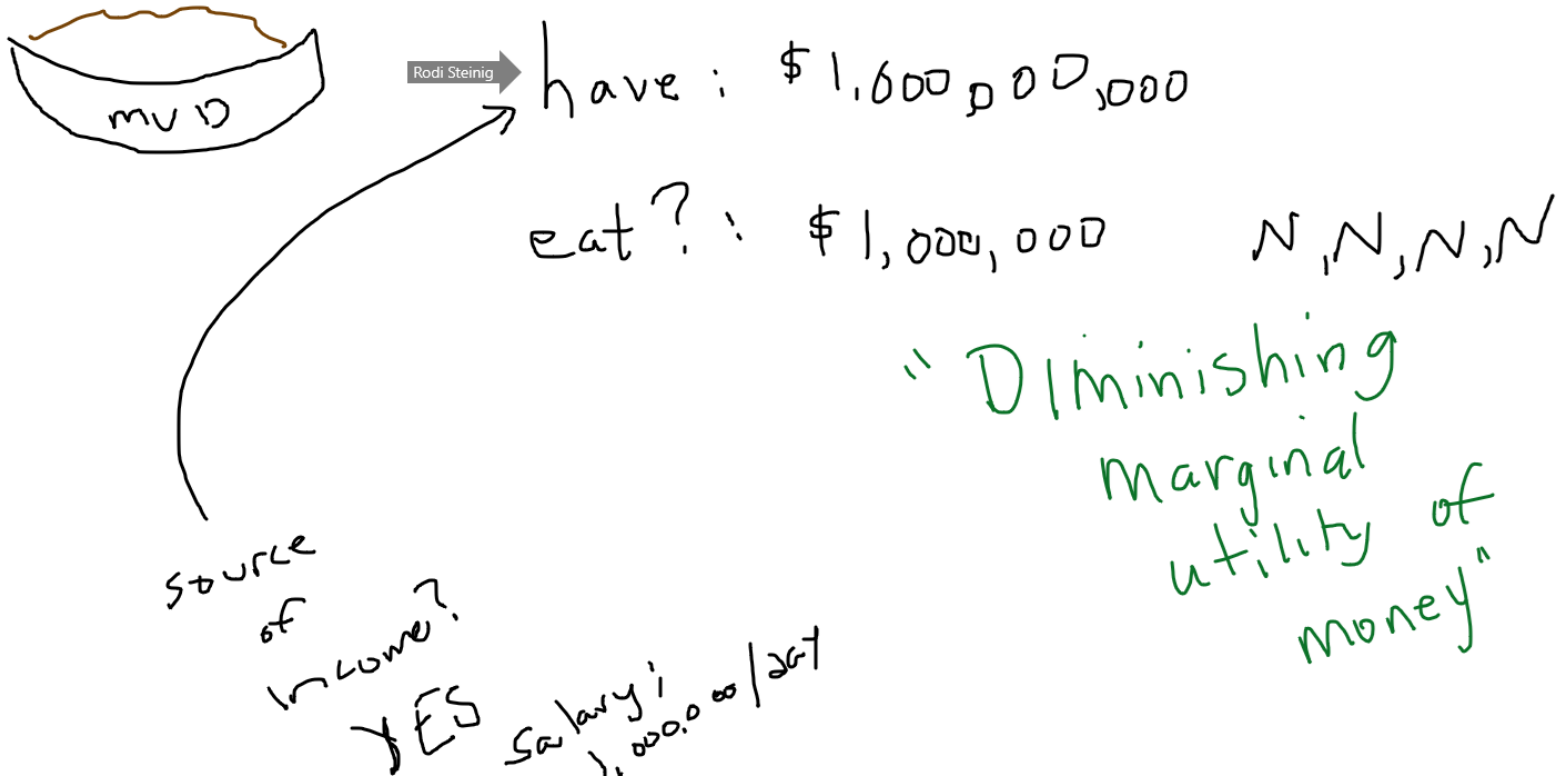
$1,000

$300,000

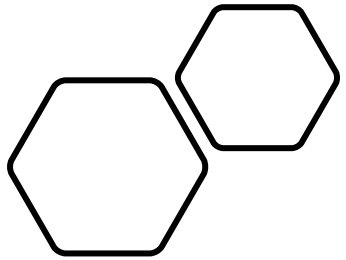
$

WOULD YOU EAT MUD?

- CONTEXT MATTERS!



“NEWCOMB’S” PROBLEM (v0.0)



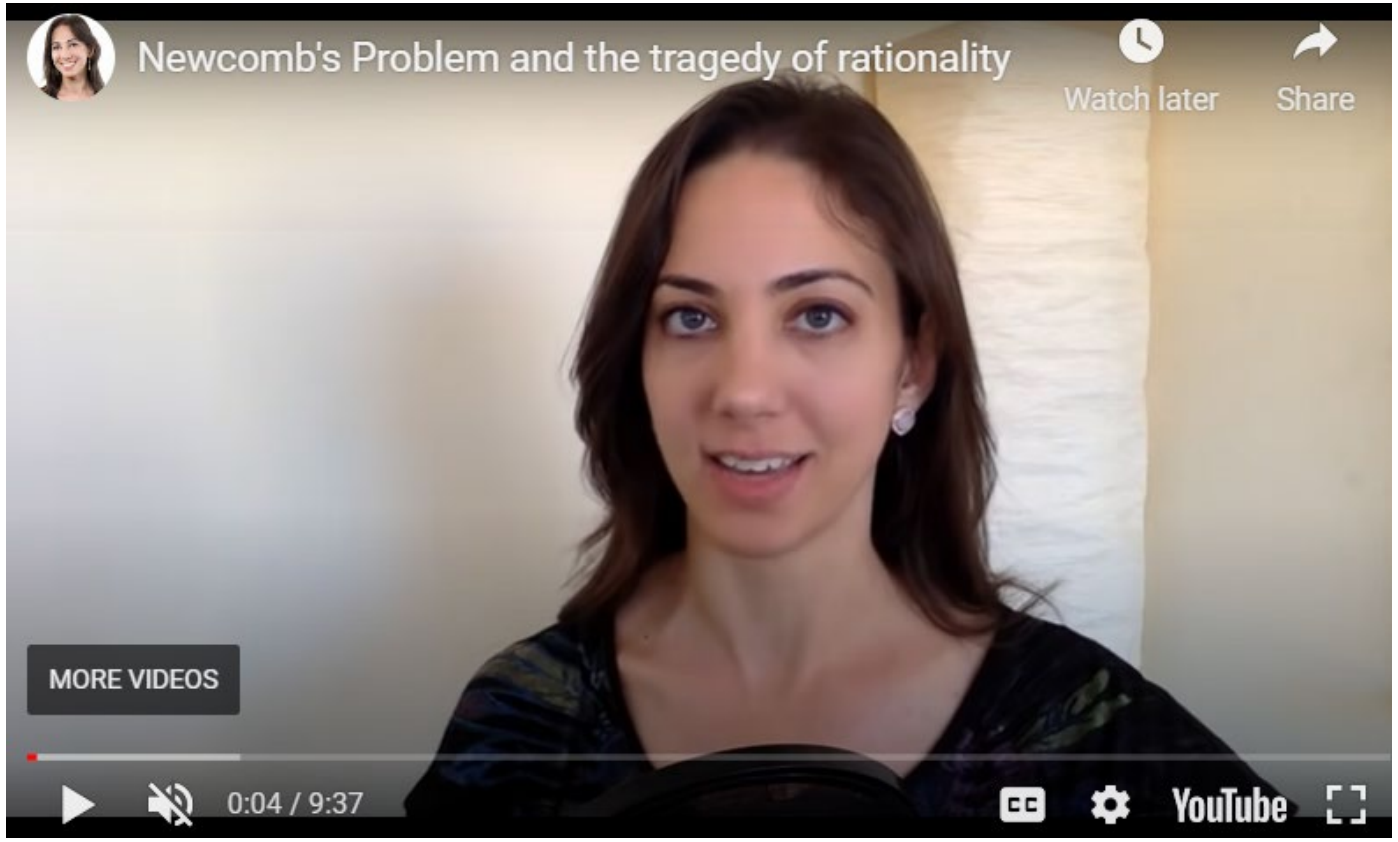
You enter a circus tent and are shown two boxes. One is clear and contains $1,000. The other is opaque and contains either nothing or $1,000,000. You can choose one or the other. Which do you choose?

“NEWCOMB’S PROBLEM” v0.0, reaction

ZALMAN: “This question seems really simple. What else is going on?”

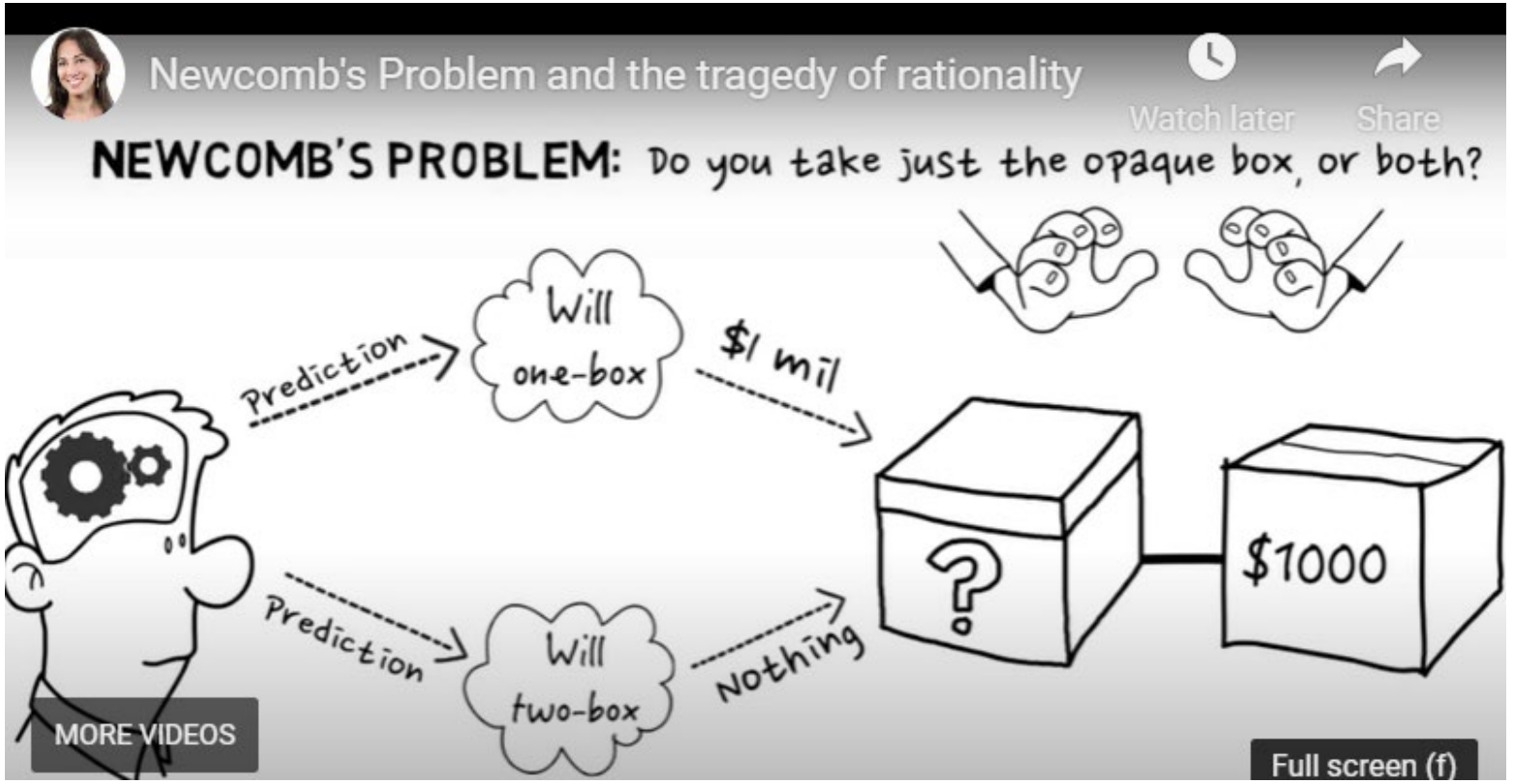
NEWCOMB'S PROBLEM – the real thing!

You enter a circus tent and are shown two boxes. One is clear and contains $1,000. The other is opaque and contains either nothing or $1,000,000. You can choose one or you can choose both. And…



Julia Galef, philosopher and skeptic

- … as you walked into the tent, there was a perfect predictor who could tell in advance with 100% certainty which box you would pick. If the predictor predicts that you would take just the opaque box, the circus tent woman put $1,000,000 in that box, but if the predictor predicts that you would take both, the circus tent woman puts nothing in the opaque box.



NEWCOMB'S PROBLEM: math

- What is a skeptic?
- Causal versus experiential decision theory
- Choice
- Free will

NEWCOMB'S PROBLEM: reaction

OBE: "I don't know why you would pick both boxes!"

NEWCOMB'S PROBLEM: reaction

KRIS: "This is frying my brain!"

NEWCOMB'S PROBLEM: reaction

ZALMAN – "It seems really simple until…"

NEWCOMB'S PROBLEM: reaction

NERO: "Is this something a mathematician would wear?"


Leonardo Pisano
Leonardo Bonacci
Leonardo Fibonacci
Fibonacci
Leonardo di Pisa
Leonardo Pisano Bigollo

STABLE MATCHING v1.0 (KIDNEY EXCHANGE)

If you needed a kidney and were offered one from a 90-year-old man, would you take it?

STABLE MATCHING v1.0: math

"KIDNEY EXCHANGE" - Decision Factors

RECIPIENTS of Kidneys

- Age
- wait time
- Risky
- Trust in doctor
- Race

DONORS of kidneys

- HLA mismatch
- KIR genotype
- Age
- Gender?
- Relation

STABLE MATCHING v2.0 (SCHOOL CHOICE)

STEP 1: Create a character whose name begins with one letter that is in your name…

STEP 2: Rank (in writing) your character's preferences for the following schools:

Wizard School

Flying School

Diplomat School

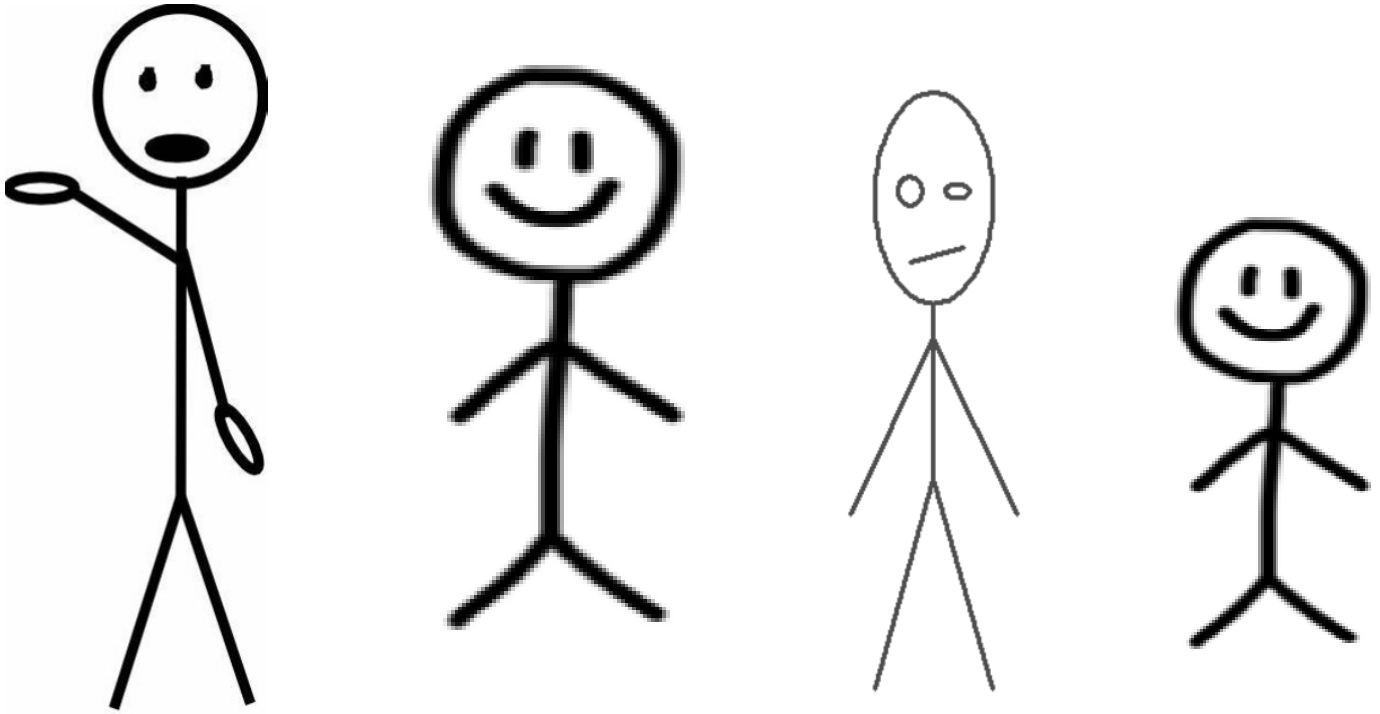
Art School

STEP 3: While you were ranking the schools, the schools were ranking "you." Let's apply the Gale-Shapley algorithm to obtain a stable match!

	SCHOOLS' PREFERENCES			
	Wizard School	**Flying School**	**Diplomat School**	**Art School**
1st choice	O	N	O	O
2nd choice	Z	Z	N	N
3rd choice	K	O	Z	Z
4th choice	N	K	K	K

Dr. Emily Riehl: Numberphile, https://www.youtube.com/watch?v=Qcv1IqHWAzg

STABLE MATCHING: reaction



	SCHOOLS' PREFERENCES			
	Wizard School	**Flying School**	**Diplomat School**	**Art School**
1st choice	O	N	O	O
2nd choice	Z	Z	N	N
3rd choice	K	O	Z	Z
4th choice	N	K	K	K

THANK YOU!

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